photometric process, with special capacities to be developed in time. I may add that I did obtain better results at the window of my house than at the laboratory at the same time, the latter being nearer the centre of the town; thus the process has done the duty it was intended for, although only once tried for this special purpose. I am looking to it as an agent specially for the examination of climate, but of course it may have many uses. This process does not aim at delicacy, but at accumulation of effect. I have not spoken of a standard; the results are only comparative, but the process may be made to supply its own standard.

12. Since writing the above it appears that by using sulphuric acid some of the fears at first entertained may be avoided, as is shown by the following extract:

B sol.	C sol. required after $2\frac{1}{2}$ hours' exposure of A to light.	C sol. required after 50 hours' exposure of A to darkness.
0·2	7·6	0·3
0·5	15·1	0·6
1·0	23·4	0·6
2·0	30·4	0·7
4·0	43·6	0·7
6·0	53·8	1·3

The temperature of the solutions exposed to light =13° C., kept in darkness =22° C. The iodine volatilized by heat was found to be so little that it might be neglected here.

The strength of solutions and the kind of acid to be used may vary. Similar results may be got by using bromide of potassium, but it is less delicate. The surface exposed and other questions require attention.

IV. "On some Elementary Principles in Animal Mechanics. No. X (concluding the series). Further illustrations of the 'Law of Fatigue.'" By the Rev. SAMUEL HAUGHTON, M.D. (Dubl.), D.C.L. (Oxon.), F.R.S., Fellow of Trinity College, Dublin. Received April 19, 1880.

The following experiments were made upon Dr. Alexander Macalister, at intervals from November, 1877, to May, 1879, and consisted in observing the lengths of time during which the extended horizontal arms, with supinated hands holding equal weights, could be held out:—

The following results were obtained, in which w denotes the weight held in the hand in pounds; and t, the time, in seconds, of holding out before fatigue stopped the experiment:

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I.

w=9·75 lbs.

No.	t.	No.	t.
1	64·0	6	64·0
2	62·0	7	61·0
3	65·0	8	57·0
4	64·0	9	76·0
5	64·0	10	68·0

Mean=64·5 seconds ± 1·1 (mean probable error).

II.

w=7·75 lbs.

No.	t.	No.	t.
1	85·0	6	76·0
2	85·0	7	81·0
3	73·0	8	90·0
4	78·0	9	80·0
5	76·0	10	85·0

Mean=80·9 seconds ± 1·1 (mean probable error).

III.

w=6·87 lbs.

No.	t.	No.	t.
1	95·0	6	90·0
2	94·0	7	99·0
3	95·0	8	93·0
4	100·0	9	91·0
5	108·0	10	90·0

Mean=95·5 seconds ± 1·2 (mean probable error).

IV.

w=5·87 lbs.

No.	t.	No.	t.
1	110·0	6	110·0
2	114·0	7	123·0
3	117·0	8	115·0
4	117·0	9	108·0
5	124·0	10	106·0

Mean=114·4 seconds ± 1·3 (mean probable error).

V.

w=4·25 lbs.

No.	*t.*	No.	*t.*
1	133·0	6	140·0
2	140·0	7	150·0
3	155·0	8	158·0
4	137·0	9	120·0
5	135·0	10	122·0

Mean=139·0 seconds ± 2·7 (mean probable error).

VI.

w=3·19 lbs.

No.	*t.*	No.	*t.*
1	164·0	6	155·0
2	169·0	7	137·0
3	145·0	8	160·0
4	170·0	9	160·0
5	170·0	10	155·0

Mean=158·5 seconds ± 2·3 (mean probable error).

VII.

w=2·56 lbs.

No.	*t.*	No.	*t.*
1	188·0	6	200·0
2	198·0	7	177·0
3	182·0	8	188·0
4	182·0	9	195·0
5	183·0	10	180·0

Mean=187·3 seconds ± 1·7 (mean probable error).

VIII.

w=0·00 lbs.

No.	*t.*	No.	*t.*
1	450·0	6	350·0
2	453·0	7	544·0
3	448·0	8	510·0
4	485·0	9	565·0
5	462·0	10	544·0

Mean=481·1 seconds ± 13·4 (mean probable error).

If we convert the mean probable errors, in seconds, of the foregoing experiments into mean probable errors in percentages, we obtain the following results:—

w.	Mean probable error.
9·75 lbs.	±1·71 per cent.
7·75 „	±1·36 „
6·87 „	±1·26 „
5·87 „	±1·13 „
4·25 „	±1·94 „
3·19 „	±1·48 „
2·56 „	±0·91 „
0·00 „	±2·78 „

This table shows the degree of credit attaching to the several groups of observations, and proves (what I did not anticipate) that the fatigue produced by small weights is as definite as that produced by large weights.

The interpretation of these observations by means of the Law of Fatigue is thus found.

When the arm is extended horizontally, the muscles of the shoulder joint keep it in position; and, at the same time, the scapula and its muscles and surroundings are raised by the reaction; so that the arm and scapula equilibrate by means of equal and opposite couples acting round the centre of the glenoid cavity. If, therefore, w denote the weight held in the hand, and α denote the weight, which, if held in the hand, would represent the sum of the couples of the arm and scapula and surroundings about the centre of the glenoid; then it is plain that the total *work* done by the muscles before fatigue stops the experiment must be proportional to $(w+\alpha)t$, where t is the whole time of the experiment. The *rate of work* is evidently proportional to

$$w+\alpha$$

Therefore, by the *Law of Fatigue* we have

$$(w+\alpha)^2t=A \quad \ldots \ (1)$$

in which α, A are unknown constants to be determined by experiment.

From the foregoing experiments, we have the eight following equations:—

$$(2).$$

$$\begin{aligned}
(\alpha+9\cdot75)^2\times\ 64\cdot5&=A.\\
(\alpha+7\cdot75)^2\times\ 80\cdot9&=A.\\
(\alpha+6\cdot87)^2\times\ 95\cdot5&=A.\\
(\alpha+5\cdot87)^2\times114\cdot4&=A.\\
(\alpha+4\cdot25)^2\times139\cdot0&=A.\\
(\alpha+3\cdot19)^2\times158\cdot5&=A.\\
(\alpha+2\cdot56)^2\times187\cdot3&=A.\\
(\alpha+0\cdot00)^2\times481\cdot1&=A.
\end{aligned}$$

From the discussion of these equations I find the following results, as in Note IX:—

α.	$\dfrac{\delta\sqrt{A}}{\Sigma(\sqrt{A})}$.
5·0	1·66 per cent.
5·5	1·49 ,,
5·6	1·40 ,,
5·7	1·39 ,,
5·8	1·41 ,,
5·9	1·50 ,,
6·0	1·60 ,,
7·0	2·31 ,,

From this, I find, by interpolation, the most probable values of a and $\sqrt{A}$ to be

$$\alpha = 5{\cdot}68 \text{ lbs.}$$

$$\sqrt{A} = 120{\cdot}31 \text{ ,,}$$

$$A = 14{,}475 \text{ ,,}$$

$$\frac{\delta\sqrt{A}}{\Sigma(\sqrt{A})} = 1{\cdot}31 \text{ per cent.}$$

From these values I obtain the following comparison of theory with observation:—

Weight.	Time of Fatigue.		Difference.	
	Calculated.	Observed.	Seconds.	Per cent.
lbs.	Secs.	Secs.		
9·75	64·5	60·8	+ 3.7	+ 5·73
7·75	80·9	80·2	+ 0·7	+ 0·87
6·87	95·5	91·9	+ 3·6	+ 3·76
5·87	114·4	108·5	+ 5·9	+ 5·41
4·25	139·0	146·8	− 7·8	− 5·31
3·19	158·5	184·0	−25·5	−13·80
2·56	187·3	213·2	−25·9	−12·10
0·00	481·1	448·7	+32·4	+ 7·21

The maximum useful work will take place when wt is a maximum, or $wt = \dfrac{Aw}{(w+\alpha)^2}$ which will be a maximum when $w=\alpha$, and the *maximum useful work* will be, if τ be the time corresponding to $w=\alpha=\dfrac{A}{4\alpha}$, $wt=\alpha\tau$, and $\tau=112{\cdot}16$ seconds.

It appears to be highly probable that, in all cases, the *most useful work* will be done when the resistance to be overcome is equal to the weight of the limb itself, estimated as applied at the same distance from the centre of motion as the resistance or weight.

I attempted, in the following manner, to check the value found for α, the equivalent of the arm and shoulder, estimated at the middle of the hand.

Dr. Macalister, closing his eyes, had his arm suspended to a balance by a string looped round the middle of the hand, and the weight of the suspended arm, allowed to hang "dead," was measured by weights placed in the opposite scale.

After many trials, I found the weight to be 5 lbs. with a possible error of 2 ozs., or $\pm 0{\cdot}125$ either way.

To find the effect of the shoulder, I measured the horizontal distance of the centre of gravity of his scapula from the centre of his glenoid cavity, and found it to be 3½ inches, while the distance of the middle of his hand from the same centre was found to be 22 inches.

The weights lifted with the scapula were thus estimated:—

Muscles.*	ozs.
Supraspinatus	1·85
Infraspinatus	3·51
Teres major	3·30
Teres minor	1·19
Subscapularis	3·88
Deltoideus (half)	5·90
Trapezius (one-third)	2·40
Serratus magnus (two-thirds)	3·92
	25·95 ozs.

To this must be added about one-third more, to represent fat, fascia, tendons, and skin, or 8·65 ozs., making altogether 34·60 ozs.

As these weights were all obtained from dead subjects, we must add somewhat more than one half,† to bring the weights up to the standard of the living subject, which makes altogether over 51·90 ozs.

Adding to this the weight of the average fresh scapula, 4·5 ozs., we have, finally for the total weight applied at the centre of gravity of the scapula, 56·40 ozs., which, when reduced to the corresponding weight held in the palm of the hand, gives $56{\cdot}4 \times \frac{3{\cdot}5}{22} = 8{\cdot}97$ ozs. $= 0{\cdot}56$ lb.

Adding this result to the weight of Dr. Macalister's arm, estimated at 22 inches from the centre of the shoulder joint, we obtain for the value of α,

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* "Principles of Animal Mechanics," p. 406.

† "Principles of Animal Mechanics," p. 45.

$$\alpha = 5{\cdot}56 \text{ lbs.} \pm 0{\cdot}125 \text{ (probable error).}$$

This result agrees remarkably well with the result $\alpha = 5{\cdot}68$ lbs., obtained by calculations founded on the Law of Fatigue, from the long series of observations detailed in this note.

V. "On the Absence of Potassium in Protagon prepared by Dr. Gamgee." By H. E. Roscoe, LL.D., F.R.S. Received April 29, 1880.

Protagon is stated by Gamgee to contain no potassium. Thudichum, on the other hand, asserts that this body contains from 0·7 to 1·6 per cent. of this metal.

In order to ascertain the quantity of potassium contained in protagon prepared by Gamgee, 5 grams of this material was completely burnt, leaving a residue of 0·177 gram, consisting chiefly of metaphosphoric acid. On precipitating the potassium in the usual way, and on decomposing the platinochloride, a residue of 0·003 gram of platinum was obtained; this corresponds to a percentage of 0·0236 of potassium on the protagon employed.

Spectroscopic experiments, made upon solutions of the chloride and phosphate of potassium, and upon a solution of the latter salt with the addition of the relative quantity of phosphoric acid present in the ash, according to the above analysis, convinced me of the correctness of the conclusions contained in my letter printed on page 113, vol. 30, of the "Proceedings of the Royal Society."

The Society adjourned over Ascension Day to Thursday, May 13.

Presents, April 8, 1880.

Transactions.

Amsterdam:—K. Akademie van Wetenschappen. Verhandelingen. Deel XIX. Afdeeling Letterkunde. Deel XII. 4to. *Amsterdam* 1879. Verslagen en Mededeelingen. Afdeeling Natuurkunde. Tweede Reeks. Deel XIV. Afdeeling Letterkunde. Tweede Reeks. Deel VIII. 8vo. *Amsterdam* 1879. Jaarboek, 1878. 8vo. Processen-Verbaal, Afdeeling Natuurkunde, 1878–9. 8vo. Elegiæ Duæ. 8vo. *Amstelodami* 1879.

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